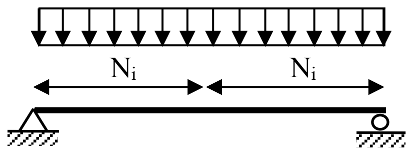
EXERCISE BAT3&4: COMPOSITE BEAM AND CONNECTION - SOLUTION

Problem 1, question 1.1

Calculating the total connection
EP calculation, therefore plastic dimensioning of the connection.

Horizontal shear force transmitted on a section of length between support (M = 0) and M_{max} , therefore :
For a simple beam and a uniformly distributed load, the shear field (critical length) corresponds to a section of length between support (M = 0) and M_{max} , i.e. the half-span: $L_i = L/2$ = 5000 mm



Horizontal shear force :

As the neutral axis of the composite section is located in the slab, the horizontal shear force to be transmitted corresponds to the plastification of the steel section:

$F_{vi,Ed} = N_{a,Rd}$ = 1401 kN

Resistance of a 16 mm stud in C 25/30 (SIA 264 § 6.1.2) :

Flexible connector? Yes because standard one (facultative, control: h_D/d = 75mm/16mm = 4.7 >4 OK)

Using E_{cm} , f_{uB} = 450 MPa, we find $P_{c,Rd}$ = 55.1 kN, and $P_{D,Rd}$ = 57.9 kN
(with SZS C5: 2018, p. 89, we have directly : P_{Rd} = 53.5 kN, but the k_E value used is slightly different).

Reduction due to profiled steel sheeting :

$$\alpha_t = \frac{0.70}{\sqrt{N_r}}\frac{b_0}{h_p}\left(\frac{h_D}{h_p} - 1\right) \leq \alpha_{t,lim}$$

In our case, the profiled steel sheeting is interrupted, see figure in the data sheet of the company, so the studs are welded directly to the steel section, so for N_r =1, $\alpha_{t,lim} = 0.75$

$$\alpha_t = \frac{0.70}{\sqrt{1}}\frac{102.5\ mm}{40\ mm}\left(\frac{75\ mm}{40\ mm} - 1\right) = 1.57 > \alpha_{t,lim} \quad \text{thus} \quad \alpha_t = 0.75$$

Nb. of studs N_i :
The number of studs required on L_i is : $N_{i,nec} = \frac{F_{vi,Ed}}{\alpha_t \cdot P_{Rd}} = \frac{1401 \cdot kN}{0.75 \cdot 55.1 kN} = 33.9$ → N_i = 34 studs.

Note: with P_{Rd} = 53.5 kN, we get more, 34.9, and without counting α_t we find less, i.e. 26.2.

Distribution of studs (C5 p. 81 or SIA 264 §7.1)
$e_2 \geq 3.5 \cdot d_D$ = 56 mm

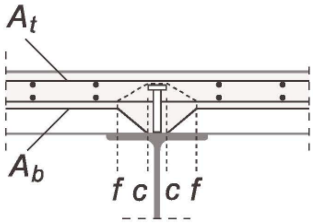
$5 \cdot d$ = 80 mm $\leq e_l \leq 6 \cdot h$ = 600 mm ≤ 800 mm

34 studs over 5000 mm gives $e_l \leq$ 147 mm, but $b_{nervure}$ =150 mm, so not good. We need to optimise further (slab thickness, etc. or put 2 studs/row near the supports).

Another possible choice: 2 studs/row, e_l = 296 mm (Warning: one would then have to recalculate with α_t = 0.6).

Problem 1, question 1.2

Longitudinal shear :
One needs to check 2 planes, c-c and f-f (see fig. 3 of SIA 264) :



Longitudinal shear force/mm between beam and slab :

$$v_{Ed} = \frac{F_{vi,Ed}}{L_i} = \frac{1401 \cdot 10^3}{5000} = 280.2\ N/mm$$

1) Verification according to plane c-c. This check is not necessary in our case according to SIA 264 § 5.1.4.3, as we have taken into account α_t to determine the number of studs. This means that the failure cone and therefore the longitudinal shear will not occur in this plane.

2) Checking according to the plane f-f
The long. shear force/mm remains unchanged for this plan.
Long. shear strength/mm :
Contribution of the transverse reinforcement: Assumption: angle of the compression strut 30°.
Bar diameter 4 mm: $A_e = 2A_b = 25.2$ mm^2

$$\nu_{Rd} = \frac{A_e}{s_f} f_{sd} \cot 30° = \frac{25.2\ mm^2}{100\ mm} \frac{500\ N/mm^2}{1.15} \cot 30° = 190 \frac{N}{mm} < \nu_{Rd,\lim} ?$$

Check against the failure limit of the compression strut (or rather the struts, given planes c-c and f-f).

This does not need to be checked usually because it is more favourable than plane c-c. But as we haven't checked the latter, we'll check it now with $L_{c,min}$ (c-c plane)
L_c = length of the potential failure surface = $2 \cdot h_D + d_k = 2 \cdot 75 + 32 = 182$ mm
(d_k : stud head diameter)

$$\upsilon_{Rd,lim} = k_c f_{cd} L_c \sin \Theta_f \cos \Theta_f = 0.6 \cdot \frac{25\ N/mm^2}{1.5} 182\ mm \cdot \sin 30° \cdot \cos 30°$$
$$= 788\ N/mm > 190\ N/mm$$

OK, it's indeed the reinforcing bar that's critical.

Contribution of the profiled sheet? No contribution, as it is interrupted over the secondary beam.
Thus we have $\upsilon_{Ed,f-f} = 280.2\ N/mm > 190\ N/mm$ → **KO**
The mesh needs to be increased to a diameter 5 mm. We then find $\nu_{Rd} = 296\ N/mm$ and the other checks remain unchanged → **OK**

Problem 1, question 1.3

Partial connection calculation
Applicable ? Composite beams for buildings under M+, ductile connectors OK

$$\frac{\boldsymbol{N}}{\boldsymbol{N_f}} \geq 1 - \frac{355\ \boldsymbol{N/mm}^2}{\boldsymbol{f_y}} \left(0{,}75\ \boldsymbol{m} - 0{,}03 \boldsymbol{L_e}\right) \geq 0{,}4 \quad \textbf{\textit{pour}}\ \boldsymbol{L_e} \leq 25\boldsymbol{m}$$

$$\frac{\boldsymbol{N}}{\boldsymbol{N_f}} \geq 1 \qquad \textbf{\textit{pour}}\ \boldsymbol{L_e} > 25\boldsymbol{m}$$

N/N_f = 1-355/235 · (0.75-0.03 · 10) = 0.32 < 0.4! The limit condition of min. 40% applies

$M_{pl,a,Rd}$ = 180 kNm (SZS C5)

$$N_{part,nec} = \frac{M_{Ed} - \frac{M_{pl,a}}{\gamma_a}}{\frac{M_{pl,b}}{\gamma_a} - \frac{M_{pl,a}}{\gamma_a}} N_{tot} = \frac{269.2 - 180}{336.6 - 180} 33.9 = 19.3 \rightarrow$$ **20** studs over 5000 mm (half span).

$$\frac{N_{part,nec}}{N_{tot}} = \frac{20}{34} = 0.59 \geq 0.4 \quad \text{OK}$$

This method significantly reduces the number of studs by 41%!

It also reduces longitudinal shear by the same proportion, i.e. around 40%, so there's no need to increase the diameter of the rebar mesh ($v_{Ed,c-c} = 168.1\ N/mm < 190\ N/mm$ **OK**).

Problem 2, question 2.1

Calculating the total connection

- Horizontal shear forces
  With the neutral axis in the steel section, we obtain the following design values for horizontal shear forces

$$F^+_{v,Ed} = N_{c,Rd} = \frac{0.85 \cdot f_{ck}}{\gamma_c} A_c = \frac{0.85 \cdot 25\ \text{N/mm}^2}{1.5} \cdot 110 \cdot 10^3\ \text{mm}^2$$
$$= 1560 \cdot 10^3\ \text{N} = 1560\ \text{kN}$$
$$F^-_{v,Ed} = N_{s,Rd} = \frac{f_{sk}}{\gamma_s} A_s = \frac{500\ \text{N/mm}^2}{1.15} \cdot 295\ \text{mm}^2 = 128 \cdot 10^3\ \text{N} = 128\ \text{kN}$$

- Stud strength d_D = 16 mm, h_D = 75 mm
  - Crushing of concrete :

$$P_{c,Rd} = \frac{0.29\, d_D^2}{\gamma_v} \sqrt{f_{ck}\, E_{cm}} = \frac{0.29\,(16\ \text{mm})^2}{1.25} \sqrt{25\ \text{N/mm}^2 \cdot 34\,400\ \text{N/mm}^2}$$
$$= 55.1 \cdot 10^3\ \text{N} = 55.1\ \text{kN}$$

  - Stud shear failure :

$$P_{D,Rd} = \frac{0.8\, f_{u,D}}{\gamma_v} \frac{\pi\, d_D^2}{4} = \frac{0.8 \cdot 450\ \text{N/mm}^2}{1.25} \cdot \frac{\pi\,(16\ \text{mm})^2}{4} = 57.9 \cdot 10^3\ \text{N} = 57.9\ \text{kN}$$

  - Reduction coefficient due to profiled sheeting (ribs parallel to the main beam) :

$$\alpha_l = 0.60 \frac{b_0}{h_p} \left(\frac{h_D}{h_p} - 1 \right) \leq 1.0$$
$$= 0.60 \frac{102.5\ \text{mm}}{40\ \text{mm}} \left(\frac{75\ \text{mm}}{40\ \text{mm}} - 1 \right) = 1.35 > 1.0 \Rightarrow \alpha_l = 1.0$$

  The design value for the shear strength of a stud is therefore $P_{Rd} = P_{c,Rd}$ = 55.1 kN

- Number of studs (total connection)
  - The number of studs for each l_i is :

$$N^+ = \frac{F^+_{v,Ed}}{P_{Rd}} = \frac{1560\ kN}{55.1\ kN} = 28.3 \Rightarrow \quad 29 \text{ dowels}$$

$$N^- = \frac{F^-_{v,Ed}}{P_{Rd}} = \frac{128\ kN}{55.1\ kN} = 2.3 \Rightarrow \quad 3 \text{ dowels}$$

  Edge spans : $N_{tot} = 2N^+ + N^- = 2 \cdot 29 + 3 = 61$ studs
  Central span: $N_{tot} = 2N^+ + 2N^- = 2 \cdot 29 + 2 \cdot 3 = 64$ studs

Note: The anchoring of end zones to take account of shrinkage forces does not generally need to be considered as it acts in the opposite direction to that due to gravitational forces (i.e. it reduces the horizontal shear force applied).

- Distribution of studs (fig. 2.1) as per above equation
  Minimum spacing between studs: 5 d_D = 5 · 16 mm = 80 mm

Maximum spacing: 800 mm or 6 h_c = 6· 100 mm = 600 mm

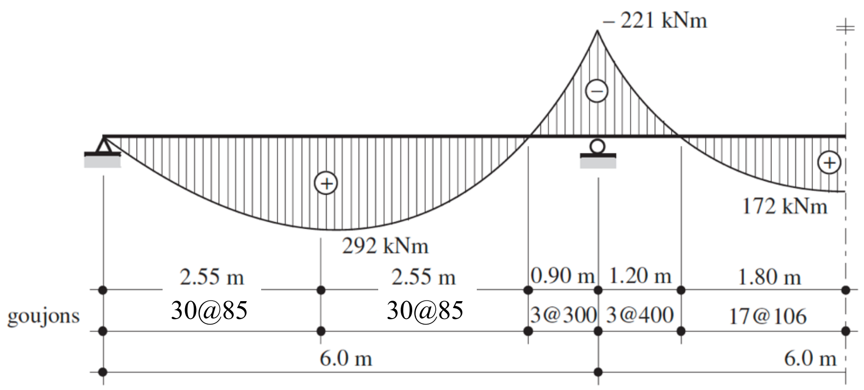


***Figure 2.1** : Diagram of redistributed moments and distribution of studs*

Note: for the central zone, we don't need a total connection, we can easily go down to a partial connection of 60%. The moment of resistance will then be :

$$M_{Rd}^{+} = M_{pl,a,Rd} + \left(M_{pl,Rd} - M_{pl,a,Rd}\right)\frac{N}{N_f} = 248\ kNm + (383\ kNm - 248)\cdot 0.6 = 330\ kNm$$

This is more than sufficient (M_{Ed} = 172 kNm).

The number of studs will then be : $N^{+} = \frac{F_{v,Ed}^{+}}{P_{Rd}} = 0.6\frac{1560\ kN}{55.1\ kN} = 17.0 \Rightarrow$ 17 studs

Problem 2, question 2.2

Longitudinal shear

- Design value for horizontal shear plastic force
  - In the zone of positive moments, the central span is decisive:

$$v_{Ed}^{+} = \frac{N}{N_f}\cdot\frac{F_{v,Ed}^{+}}{l_2^{+}} = 0.6\cdot\frac{1560\cdot 10^3\ \text{N}}{1800\ \text{mm}} = 520\ \text{N/mm}$$

  - In the area of negative moments, the edge span is decisive:

$$v_{Ed}^{-} = \frac{F_{v,Ed}^{-}}{l_1^{-}} = \frac{128\cdot 10^3\ \text{N}}{900\ \text{mm}} = 142\ \text{N/mm}$$

- Horizontal shear force per unit length and per plane g-g
  - Positive moments :

$$v_{Ed(g-g)}^{+} = v_{Ed}^{+}\frac{b_{eff,1} - b}{2b_{eff,1}} = 520\ \text{N/mm}\cdot\frac{1275\ \text{mm} - 280\ \text{mm}}{2\cdot 1275\ \text{mm}} = 203\ \text{N/mm}$$

  - Negative moments :

$$v_{Ed(g-g)}^{-} = v_{Ed}^{-}\frac{b_{eff,2} - b}{2b_{eff,2}} = 142\ \text{N/mm}\cdot\frac{750\ \text{mm} - 280\ \text{mm}}{2\cdot 750\ \text{mm}} = 44.5\ \text{N/mm}$$

- Longitudinal shear strength per unit length
  - Contribution of the transverse reinforcement with an increased diameter of 6 mm, s_f = 100 mm:

$$v_{Rd} = \frac{A_e}{s_f}f_{sd}\cot\Theta_f = \frac{28.3\ \text{mm}^2}{100\ \text{mm}}\cdot\frac{500\ \text{N/mm}^2}{1.15}\cdot\cot 30^\circ = 213\ \text{N/mm}$$

with

$$A_e = \frac{\pi\,\phi^2}{4} = \frac{\pi (6\text{ mm})^2}{4} = 28.3\text{ mm}^2$$
$$\Theta_f = 30^\circ\ \text{(choix)}$$

- Limit due to compression strut failure :

$$v_{Rd,lim} \leq k_c\, f_{cd}\, L_c\, \sin\Theta_f\, \cos\Theta_f = 0.6 \cdot \frac{25\text{ N/mm}^2}{1.5} \cdot 60\text{ mm} \cdot \sin 30^\circ \cdot \cos 30^\circ = 259.8\text{ N/mm}$$

with $k_c = 0.6$
$L_e = h_c = 60$ mm (g-g plane)

The design value of the longitudinal shear strength is therefore v_{Rd} = 213 N/mm since v_{Rd} = 213 N/mm < $v_{Rd,\lambda\iota\mu}$ = 259.8 N/mm.

- Checks (section g-g)
  - Positive moments :

    v_{Ed}^+ = 203 N/mm < v_{Rd} = 213 N/mm OK, but would be KO with rebars of diam. 4mm !
  - Negative moments :

    v_{Ed}^+ = 44.5 N/mm < v_{Rd} = 213 N/mm OK